A Study of Systematic Variation of Asymmetric Parameter on NpNn in Nuclear Mass Region

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Abstract:
The paper examines even-even nuclei's collective nuclear structure through investigation of essential observables under proton and neutron number multiplication (NpNn). An analysis of the asymmetry parameter (γ) variation occurs across different (N, Z) regions with emphasis on nuclei that fall within N = 28–50 and Z = 28–50 as well as N = 40–66 and Z = 40–50. The research findings show intricate relationships controlling the collectivity in nuclei and their shapes because NpNn follows a continuous trend in which specific patterns appear linked to the shell structure's defining magic numbers. Nuclei display behavior which appears similar to both spherical U(5) and axially deformed SU(3) patterns while total deformation shapes their configuration. This research contributes essential knowledge regarding nuclear interactions while developing insights about nuclear shape evolution for use as a base in future studies of nuclear physics and nuclear structure principles.

Keywords : *valence necleons, asymmetry parameter* $(\boldsymbol{\gamma_0})$, *shell structure, SU(5), SU(3)*

Introduction

The collective behavior of atomic nuclei can be effectively explored using variables such as neutron number (N), proton number (Z), the total boson number (NB), and especially the product of valence protons and neutrons (NpNn). These parameters provide valuable insights into the interactions that shape nuclear structure. Over the years, a wide range of studies has been conducted to investigate how collective motion, deformation, and other structural characteristics evolve in relation to NpNn. For instance, Martinou et al. (2020) emphasized the crucial contribution of valence nucleons in shaping nuclear properties, while Bhattacharya et al. (2023) highlighted the robustness of level structures in semi-magic isotones and isotopes. Similarly, Möller et al. (2016) demonstrated that both protons and neutrons play a pivotal role in driving nuclear deformation.

Within the framework of the Interacting Boson Model (IBM), Zyriliou et al. (2022) showed that the structure of nuclei is governed by the total number of bosons (NB). This idea laid the foundation for the concept of F-spin multiplets, which was later elaborated by Zhao et al. (2021). Furthermore, Wang et al. (2023) observed that the excitation energy of the first 2^+ state shows a smooth and consistent variation with NpNn. Other investigations, including those by Bao et al. (2020), have examined how various observables related to collectivity and deformation exhibit systematic trends with respect to the NpNn product.

Gupta and Hamilton (2022) identified a linear correlation between the inverse of the rotational coefficient (1/α) and NpNn, where α represents the contribution to the rotational energy in the SU(3) symmetry limit of the IBM, as described by the equation:

E([N](λ,μ)KLM)=αL(L+1)+βC(λ,μ)E([N](\lambda, \mu)KLM) = \alpha L(L+1) + \beta C(\lambda, \mu)E([N](λ,μ)KLM)=αL(L+1)+βC(λ,μ)

This relationship suggests that rotational behavior in nuclei can be quantified directly through the NpNn scheme. Moreover, transition probabilities such as B(E2; $2^+_1 \rightarrow 0^+_1$) have also been linked to this product. In another detailed analysis, Walker and Podolyák (2023) revealed a clear dependence of γ–g B(E2) ratios on NpNn values across different shell regions—specifically for proton numbers between Z = 50 and 82 and neutrons below or above N = 82. A review by Wang et al. (2023) further outlined the gradual evolution of nuclear structure driven by the NpNn framework. To enhance this model, a modified approach involving the P-factor was introduced by Walker and Podolyák (2023), adding further depth to the analysis of nuclear collectivity.

In this study, we extend the understanding of valence nucleons and holes in relation to nuclear structure by analyzing trends based on the NpNn product. While Bao et al. (2020) explored regions with mass numbers around A = 100, 130, 150 (for Z < 64 and Z > 64), and A = 190, our research concentrates on the regions defined by N = 28–50 and Z = 28–50, as well as N = 50–82 and Z = 28–50. These regions are of particular interest because they span the well-known magic numbers, which mark significant energy gaps in the shell model's single-particle level scheme.

To examine systematic variations, we calculated the asymmetry parameter (γ) across the range $28 \leq Z \leq 50$ and $28 \leq N \leq 50$. The dataset was divided into four quadrants, and the γ values were plotted against NpNn to reveal potential trends in the collective behavior of nuclei within this segment of the nuclear chart.

Calculation of Asymmetric Parameter (γ_0)

The value of γ_0 can be evaluated using the experimental energies $E2_2^+$ and $E2_1^+$ states (Data, 2000). The energy ratio $R\gamma = E_{2\gamma} / E_{2g}$ and γ is:

$$\boldsymbol{\gamma_0} = \left(\frac{1}{3}\right) \sin^{-1}\left[\left(\frac{9}{8}\right) 1 - \left(\frac{R_\gamma - 1}{R_\gamma + 1}\right)\right]^{1/2} . \; \text{... eqn (1)}$$

Asymmetric Parameter Equation.

It can be evaluated using: (a) The energy ratio $R_4 = (E_{4g}/E_{2g})$ but only the nuclei with $2.8 \leq R_4 \leq 3.33$ will be allowed (Sharma et al.)[11, 12]. (b)The B(E2) values which are very small and available with uncertainties.Therefore the values from energy ratio Rγ are more reliable.

Result and discussions

The variation of γ versus NpNn product for quadrant-I for $28 \leq Z \leq 50$ and $28 \leq N \leq 50$ has been shown in Fig. 1. There is smooth dependence of γ with NpNn. The γ decreases fast while NpNn increase from 0 to 10. There is strong dependence of γ on NpNn because all data points are merged in a single curve, except Kr (NpNn=20). This means that the nuclei are becoming either spherical (U(5)) or axially deformed (SU(3)), depending on the total deformation. Conversely, it implies that the system is moving toward a more stable quadrupole-deformed shape, favoring an SU(3) (axially deformed) structure.

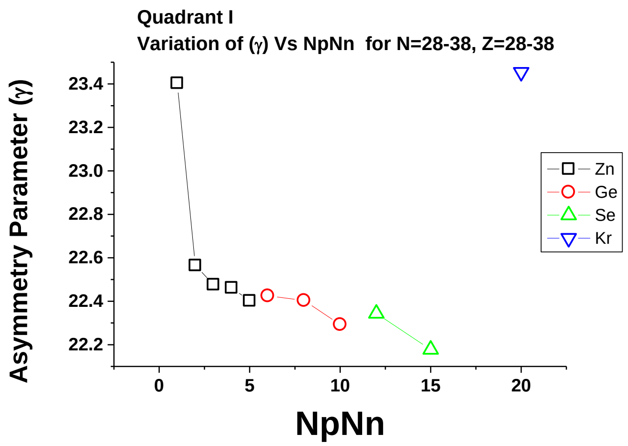

Fig 1. The variation of asymmetry parameter (γ) versus NpNn for N=28 to 38 and Z=28 to 38 (Zn to Kr) quadrant I.

The variation of γ versus NpNn for N=40 to 50 and Z=40 to 50 quadrant III is shown in Fig 2. For Zr, the γ is increasing from 22.4^0 to 23.5^0 while NpNn increasing from 0 to 20. For Mo, the γ decrease while NpNn is increasing from 0 to 8. There is weak dependence of γ on NpNn.

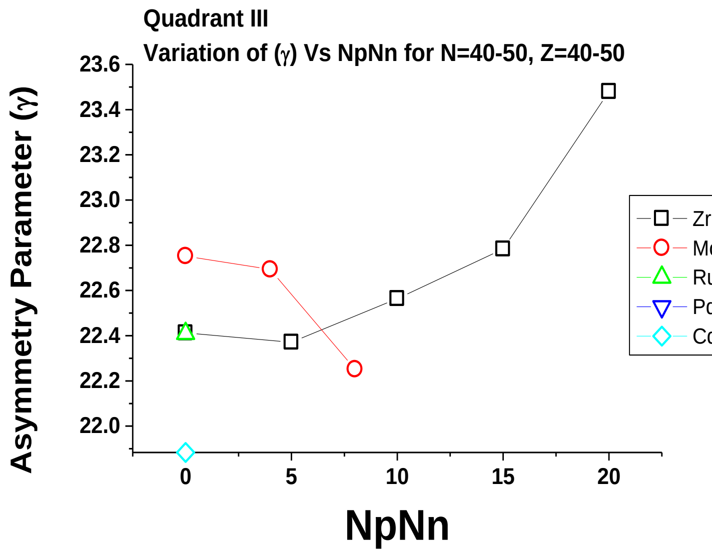

Fig. 2. The variation of asymmetry parameter (γ) versus NpNn for N=40 to 50 and Z=40 to 50 (Zr to Cd) quadrant III.

The variation of γ versus NpNn for N=40 to 50 and Z=28 to 38 isotopes of quadrant IV is shown in Fig. 3. There is weak dependence of γ on NpNn.

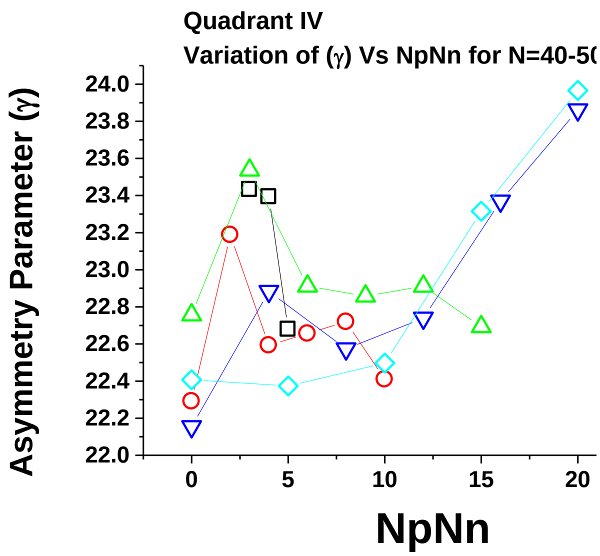

Fig. 3 The variation of asymmetry parameter (γ) versus NpNn for N=40 to 50 and Z=28 to 38 (Ni to Kr) quadrant IV.

The variation of γ versus NpNn for N=50 to 66 and Z=28 to 38 (Ge to Sr) quadrant I is shown in Fig. 4. There is strong dependence of γ on NpNn because all data points are merged in a single curve.

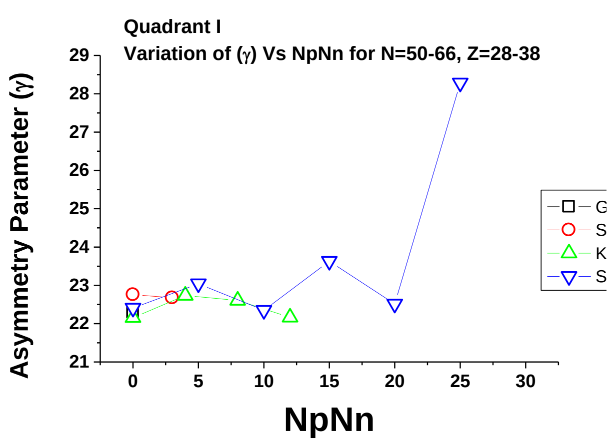

Fig. 4 The variation of asymmetry parameter (γ) versus NpNn for N=50 to 66 and Z=28 to 38 (Ge to Sr) quadrant I

The variation of γ versus NpNn for N=50 to 66 and Z=40 to 50 (Zr to Sn) quadrant II is shown in Fig. 5. There is strong dependence of γ on NpNn because all data points are merged in a single curve, except some scattering at NpNn=15-25.

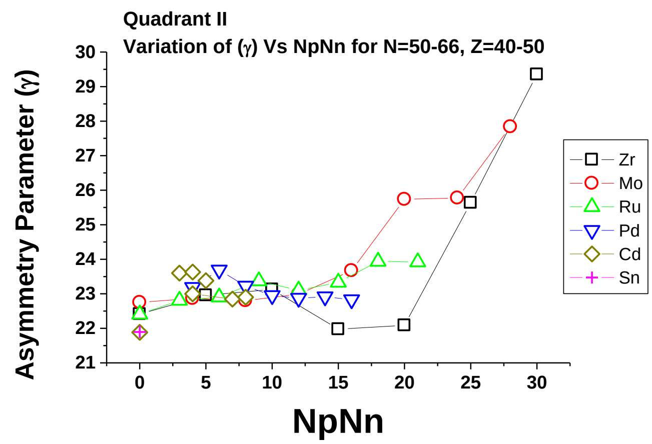

Fig. 5. The variation of asymmetry parameter (γ) versus NpNn for N=50 to 66 and Z=40 to 50 (Zr to Sn) quadrant II.

The variation of γ versus NpNn for N=66 to 82 and Z=40 to 50 quadrant III is shown in Fig. 6. There is no dependence of γ on NpNn.

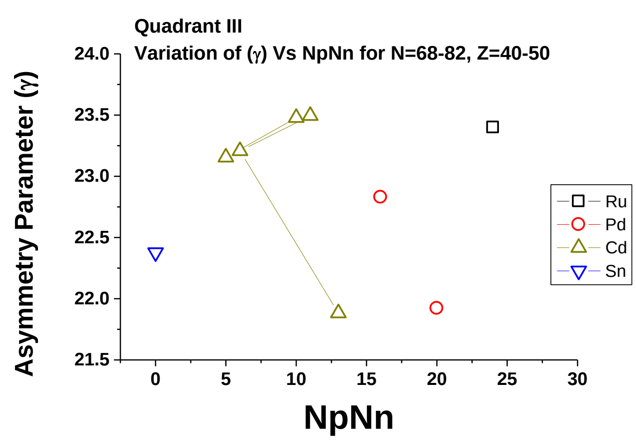

Fig. 6 The variation of asymmetry parameter (γ) versus NpNn for N=66 to 82 and Z=40 to 50 (Ru to Sn) quadrant III.

Conclusion

This study provides significant understandings into the complex interactions between valence nucleons and the collective nuclear structure in even-even nuclei. By examining the systematic dependence of nuclear observables on the product of proton and neutron numbers (NpNn), the study highlights how these factors influence the asymmetry parameter (γ) and overall nuclear deformation. The findings demonstrate that various quadrants in the (N, Z) chart exhibit distinct patterns that correlate with underlying shell structure, mainly around magic numbers, which serve as essential reference points in understanding nuclear stability and deformation. The smooth dependence of γ on NpNn, as well as the observed variations across different quadrants, sheds light on the dynamics of nuclear influences and the evolution of nuclear shapes. This work therefore contributes to a deeper understanding of nuclear structure and may pave the way for further research into the basics of nuclear physics, particularly in discovering non-standard arrangements within the context of modern theoretical frameworks

REFERRENCES

Bao, M., Zong, Y., Zhao, Y., & Arima, A. (2020). Local relations of nuclear charge radii. *Physical Review C*, *102*(1), 014306.

Bhattacharya, S., Tripathi, V., Tabor, S., Volya, A., Bender, P., Benetti, C., Carpenter, M., Carroll, J., Chester, A., & Chiara, C. (2023). β^- decay of neutron-rich ^{45} Cl at magic number N= 28. *arXiv preprint arXiv:2305.14466*.

Data, E. N. R. (2000). National Nuclear Data Center. *Brookhaven National Laboratory (http://www. nndc. bnl. gov/exfor/exfor00. htm) and International Atomic Energy Agency, Nuclear Data Services (http://www-nds. iaea. org/exfor/exfor. htm).*

Gupta, J., & Hamilton, J. (2022). Analysis of the SU (3) symmetry versus rotor model. *Physical Review C*, *105*(6), 064312.

Martinou, A., Bonatsos, D., Minkov, N., Assimakis, I., Peroulis, S., Sarantopoulou, S., & Cseh, J. (2020). Proxy-SU (3) symmetry in the shell model basis. *The European Physical Journal A*, *56*, 1-18.

Möller, P., Sierk, A. J., Ichikawa, T., & Sagawa, H. (2016). Nuclear ground-state masses and deformations: FRDM (2012). *Atomic Data and Nuclear Data Tables*, *109*, 1-204.

Sharma, S., Jawa, I. M., & Kumar, R. Tests of Rigid Triaxiality for Light Te-Sm Even-Even Nuclei from Rigid Triaxial Asymmetric Rotor Model.

Walker, P. M., & Podolyák, Z. (2023). Nuclear isomers. In *Handbook of Nuclear Physics* (pp. 487-523). Springer.

Wang, M., Zhang, Y., Zhou, X., Zhou, X., Xu, H., Liu, M., Li, J., Niu, Y., Huang, W., & Yuan, Q. (2023). Mass measurement of upper fp-shell N= Z-2 and N= Z-1 nuclei and the importance of three-nucleon force along the N= Z line. *Physical review letters*, *130*(19), 192501.

Zhao, R., Grofe, A., Wang, Z., Bao, P., Chen, X., Liu, W., & Gao, J. (2021). Dynamic-then-static approach for core excitations of open-shell molecules. *The Journal of Physical Chemistry Letters*, *12*(31), 7409-7417.

Zyriliou, A., Mertzimekis, T., Chalil, A., Vasileiou, P., Mavrommatis, E., Bonatsos, D., Martinou, A., Peroulis, S., & Minkov, N. (2022). A study of some aspects of the nuclear structure in the even–even Yb isotopes. *The European Physical Journal Plus*, *137*(3), 352.